

Homework 8 – Section 12A/12B/12C

17.2.1 Voting Methods - Winning by Plurality

Identify elements of a preference table

1. Robinson Elementary School students are ranking their favorite school lunches. The choices are pizza (P), chicken fingers (C), hamburgers (H), and spaghetti (S). The ballots have been combined into the following preference table.

	112	63	85	103	51	98
1st choice	<i>P</i>	<i>S</i>	<i>S</i>	<i>P</i>	<i>H</i>	<i>C</i>
2nd choice	<i>S</i>	<i>C</i>	<i>H</i>	<i>C</i>	<i>P</i>	<i>H</i>
3rd choice	<i>C</i>	<i>P</i>	<i>P</i>	<i>H</i>	<i>S</i>	<i>P</i>
4th choice	<i>H</i>	<i>H</i>	<i>C</i>	<i>S</i>	<i>C</i>	<i>S</i>

What percentage of students picked spaghetti as their first choice? Round your answer to one decimal place.

How many students picked pizza as their first choice?

2. Given the preference table below, how many voters have candidate B as their second choice? And candidate D as their third choice

	13	64	19	110	33	15
1st choice	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>A</i>	<i>D</i>
2nd choice	<i>D</i>	<i>C</i>	<i>A</i>	<i>C</i>	<i>B</i>	<i>B</i>
3rd choice	<i>B</i>	<i>D</i>	<i>D</i>	<i>A</i>	<i>D</i>	<i>C</i>
4th choice	<i>C</i>	<i>A</i>	<i>B</i>	<i>B</i>	<i>C</i>	<i>A</i>

Answers

1. 28.89%, 215 students
2. 48 voters ,116 voters

Determine the winner of an election using the plurality method

1. Suppose the board members of a company are deciding where to move their corporate office. They are considering moving to Florida (F), Maine (M), Arkansas (A), or California (C). The board conducted a poll among the employees of the company. The preference table below shows the results.

	50	40	45	20	30	20
1st	<i>A</i>	<i>M</i>	<i>M</i>	<i>M</i>	<i>F</i>	<i>C</i>
2nd	<i>C</i>	<i>C</i>	<i>C</i>	<i>A</i>	<i>C</i>	<i>M</i>
3rd	<i>M</i>	<i>A</i>	<i>F</i>	<i>C</i>	<i>M</i>	<i>F</i>
4th	<i>F</i>	<i>F</i>	<i>A</i>	<i>F</i>	<i>A</i>	<i>A</i>

Assuming that the board will use the plurality method to determine where to move the office, how many first-choice votes does the winning option receive?

What is the minimum number of additional votes that California needs to win the election?

2. Consider the preference table below.

	23	12	15	10	15	5
1st	<i>A</i>	<i>A</i>	<i>C</i>	<i>B</i>	<i>C</i>	<i>B</i>
2nd	<i>B</i>	<i>C</i>	<i>A</i>	<i>A</i>	<i>B</i>	<i>C</i>
3rd	<i>C</i>	<i>B</i>	<i>B</i>	<i>C</i>	<i>A</i>	<i>A</i>

Using the plurality method, how many first-choice votes would the third-place finisher need to add to win (without a tie)?

3. Consider the preference table below for an election.

	26	31	<i>x</i>	17	18	24
1st	<i>A</i>	<i>M</i>	<i>C</i>	<i>M</i>	<i>F</i>	<i>C</i>
2nd	<i>C</i>	<i>C</i>	<i>M</i>	<i>A</i>	<i>C</i>	<i>M</i>
3rd	<i>M</i>	<i>A</i>	<i>F</i>	<i>C</i>	<i>M</i>	<i>F</i>
4th	<i>F</i>	<i>F</i>	<i>A</i>	<i>F</i>	<i>A</i>	<i>A</i>

The election officials will use the plurality method to determine the outcome of the election. What is the smallest value for x that will ensure C will win the election?

Answers

1. Maine wins with 105 votes. California needs 86 more votes to win
2. B needs 21 more votes to win.
3. Both Atlanta and Ausin have 85 votes, resulting in a run-off.
4. $x = 25$

Determine the winner of an election using the Borda count method

1. Suppose CoffeeMan, LLC is looking for a city in which to locate their headquarters. Consider the voting preference table below.

	7	1	4
1st	Seattle	Emporia	Tacoma
2nd	Tacoma	Tacoma	Emporia
3rd	Emporia	Seattle	Seattle

Using the election data and the Borda count method, how many voting points does the second-place finisher receive?

2. Based on the preference table below, how many voting points does the winning candidate receive, using the Borda count method?

	32	41	50
1st	<i>A</i>	<i>B</i>	<i>C</i>
2nd	<i>B</i>	<i>C</i>	<i>A</i>
3rd	<i>C</i>	<i>A</i>	<i>B</i>

3. Consider the preference table below. Which list shows the candidates, A, B, C, or D, in the correct order of voting points from highest number of points to lowest number of points, under the Borda count method?

	21	12	6	47	9
1st	A	B	A	D	A
2nd	C	C	C	C	B
3rd	B	D	D	A	D
4th	D	A	B	B	C

- (C, D, A, B)
- (D, C, A, B)
- (A, C, D, B)
- (D, A, C, B)

4. Consider the preference table below. If the Borda count method is used to determine the winner, how many voting points does the last-place finisher have in this election?

	30	12	6	25
1st	Ronny	Jill	Ronny	Carlos
2nd	Jill	Carlos	Carlos	Ronny
3rd	Bobby	Bobby	Jill	Bobby
4th	Carlos	Ronny	Bobby	Jill

5. Consider the preference table below. Which candidate, A, B, C, or D wins the following election if the Borda count method is used?

	21	12	6	47	9
1 st Choice	A	B	A	D	A
2 nd Choice	C	C	C	C	B
3 rd Choice	B	D	D	A	D
4 th Choice	D	A	B	B	C

6. Suppose four people are running for positions in the Graduate Student Council. The council will decide the election using the Borda count method. Consider the preference table below.

	21	12	6	47
1st	Jill	Bobby	Ronny	Carlos
2nd	Ronny	Carlos	Carlos	Ronny
3rd	Bobby	Jill	Jill	Bobby
4th	Carlos	Ronny	Bobby	Jill

The two candidates who receive the most voting points will be on the council. Which two students will be elected?

- Bobby
- Carlos
- Jill
- Ronny

Answers

1. 26 points
2. 264 points
3. (C, D, A, B)
4. 140 points
5. candidate C
6. Carlos and Ronny

17.2.2 Comparison Voting Methods

Decide the winner of an election using the plurality with elimination method

1. Consider the voting preference table below.

	21	42	36	47	29	51
1st	<i>A</i>	<i>B</i>	<i>A</i>	<i>D</i>	<i>A</i>	<i>C</i>
2nd	<i>D</i>	<i>C</i>	<i>C</i>	<i>C</i>	<i>B</i>	<i>B</i>
3rd	<i>B</i>	<i>D</i>	<i>D</i>	<i>A</i>	<i>D</i>	<i>D</i>
4th	<i>C</i>	<i>A</i>	<i>B</i>	<i>B</i>	<i>C</i>	<i>A</i>

2. Consider the voter preference table below.

	23	12	15	10	20	9
1st	Ally	Twyla	Twyla	Jon	Bill	Jon
2nd	Jon	Jon	Ally	Twyla	Twyla	Ally
3rd	Bill	Ally	Bill	Bill	Jon	Twyla
4th	Twyla	Bill	Jon	Ally	Ally	Bill

Using the plurality with elimination method, which of the following situations would produce a majority winner in the second round?

- Ally receives 20 more first-choice votes and no other additional votes exist
- Bill receives 25 more first-choice votes and no other additional votes exist
- None of these choices
- Twyla receives 16 more first-choice votes and no other additional votes exist

3. Consider the voting preferences in the table below.

	23	12	15	10	20	9
1st	Aly	Twyla	Twyla	Jon	Bill	Jon
2nd	Jon	Jon	Aly	Twyla	Twyla	Aly
3rd	Bill	Aly	Bill	Bill	Jon	Twyla
4th	Twyla	Bill	Jon	Aly	Aly	Bill

Using the plurality with elimination method, which of the choices leads the election at the end of the second round, and with how many first-choice votes?

- Ally leads at the end of the second round with 40 first-choice votes
- Twyla leads at the end of the second round with 37 first-choice votes
- Ally leads at the end of the second round with 32 first-choice votes
- Twyla leads at the end of the second round with 32 first-choice votes

4. Consider the voting preference table below.

	21	12	26	47	19	11
1st	A	D	C	C	B	D
2nd	B	A	D	D	D	A
3rd	C	C	A	B	A	B
4th	D	B	B	A	C	C

Using the plurality with elimination method, how many first-choice votes will the winner have at the end of the last round?

5. Consider the preference table below.

	3	4	5
1st	A	B	C
2nd	B	C	A
3rd	C	A	B

- A wins in the first round
- B wins in the first round
- C wins in the first round
- A wins in the second round
- B wins in the second round
- C wins in the second round

6. Consider the voting preference table below.

	23	12	15	10	20	9
1st	Ally	Twyla	Twyla	Jon	Bill	Jon
2nd	Jon	Jon	Ally	Twyla	Twyla	Ally
3rd	Bill	Ally	Bill	Bill	Jon	Twyla
4th	Twyla	Bill	Jon	Ally	Ally	Bill

Using the plurality with elimination method, how many first-choice votes will the winner have at the end of the last round?

Answers

1. C wins with 140 votes
2. Twyla receives 16 more first-choice votes and no other additional votes exist
3. Twyla leads at the end of the second round with 37 first-choice votes
4. C receives 73 first-choice votes in the first round, which is a majority
5. B wins in the second round
6. Twyla wins the election in the third round with 57 first-choice votes.

Select the winner of an election using the pairwise comparison method

1. Consider the voter preference table below.

	21	12	6	47	9
1st	A	B	A	D	A
2nd	D	C	C	C	B
3rd	B	D	D	A	D
4th	C	A	B	B	C

- Choice A wins the election
- Choice B wins the election
- Choice C wins the election
- Choice D wins the election
- There is a tie between A and D
- There is a tie between B and C
- There is a three-way tie among B, C, and D

Using the pairwise comparison method, how many more points does D receive than B?

2. Consider the preference table below.

	32	54	41	37	28
1st	A	B	B	B	D
2nd	C	C	C	A	C
3rd	B	A	D	C	B
4th	D	D	A	D	A

If the pairwise comparison method is used to determine the election results, how many points does choice C receive?

3. Consider the preference table below.

	3	4	5
1st	A	B	C
2nd	B	C	A
3rd	C	A	B

Use the pairwise comparison method between B and C. How many votes does the winner of this comparison receive?

4. If an election with four candidates is decided using the pairwise comparison method, which of the following situations is NOT possible?

- A candidate loses with 1 point
- A candidate loses with 2 points
- A candidate wins with 3 points
- A candidate wins with 4 points

5. Consider the preference table below from an election.

	30	12	6	25
1st	Ronny	Jill	Ronny	Carlos
2nd	Jill	Carlos	Carlos	Ronny
3rd	Bobby	Bobby	Jill	Bobby
4th	Carlos	Ronny	Bobby	Jill

If the pairwise comparison method is used to determine the results of the election, which candidate would win?

- Bobby wins the election
- Jill wins the election
- Ronny wins the election
- Ronny and Jill tie
- Bobby and Carlos tie
- There is a three-way tie among Carlos, Jill, and Ronny
- There is a three-way tie among Bobby, Jill, and Ronny

6. Consider the voter preference table below.

	12	13	17	21	27	11
1st	A	B	E	C	A	C
2nd	D	E	A	D	E	E
3rd	B	C	C	A	B	D
4th	C	D	D	B	D	A
5th	E	A	B	E	C	B

Using the pairwise comparison methods, how many points (from winning pairwise comparisons) does C have?

Answers

1. Choice D wins the election, and has 3 more points than B since B has no votes
2. B wins with 3 points, and C came in second with 2 points
3. 7 votes
4. In a four way race, say between A,B,C, and D, there are six pairwise comparisons, AB, AC, AD, BC, BD, and CD. Each candidate is compared with three other candidates. The maximum number of times a candidate can win in a comparison is 3 times. Therefore, a candidate cannot receive more than 3 points.
5. There is a three-way tie among Carlos, Jill, and Ronny
6. In total, C earned 1 point

17.3.1 Criterion of Voting Methods - Majority, Head to Head count, and Monotonicity

Determine a voting systems fairness using the majority criterion

1. A group of 200 leaders in a city are voting on a temporary replacement for the mayor. Leaders are choosing from the following four candidates: Jones, Brown, Lewis, and Davis. Four separate votes for the temporary mayor were as follows:

	37	43	81	39
1st	Jones	Lewis	Davis	Jones
2nd	Brown	Jones	Jones	Lewis
3rd	Lewis	Brown	Lewis	Brown
4th	Davis	Davis	Brown	Davis

Which candidate is the winner using the majority criterion?

- Lewis
- Davis
- Brown
- Jones
- Cannot be determined

2. A person gave a dog to an animal rights group. The group is deciding upon a name, choosing from the following four names: Pox, Zac, Gus, and Kid. The results from four rounds of preferences for the name were:

	12	14	38	15	9
1st	Pox	Kid	Gus	Pox	Gus
2nd	Zac	Gus	Kid	Gus	Gus
3rd	Gus	Pox	Zac	Kid	Zac
4th	Kid	Zac	Pox	Zac	Kid

Using the plurality method, which name will be selected? Does this violate the majority criterion?

- Gus, no
- Gus, yes
- Pox, no
- Pox, yes
- Kid, yes
- Zac, yes
- Zac, no

3. A group of scientists is getting together for a conference. They are deciding upon a location, choosing from the following four cities: Aspen, Omaha, Butte, and Tulsa. The votes from four rounds of voting for where to hold the conference were as follows:

	53	21	11	15
1st	Aspen	Tulsa	Omaha	Aspen
2nd	Omaha	Omaha	Aspen	Tulsa
3rd	Butte	Aspen	Tulsa	Butte
4th	Tulsa	Butte	Butte	Omaha

- Aspen, yes
- Aspen, no
- Butte, yes
- Omaha, yes
- Omaha, no
- Tulsa, no

4. A state math society is voting on the main design of a new flag. The flag committee has narrowed the designs to four: Polygon, Summation, Integral, or Conic. A ballot was mailed out to the membership four times trying to determine the design the most members wanted. The results of the rankings are summarized below.

	165	382	431	297
1st	Polynomial	Conic	Integral	Integral
2nd	Summation	Polynomial	Summation	Polynomial
3rd	Conic	Summation	Conic	Summation
4th	Integral	Integral	Polynomial	Conic

Using the plurality method, which design will be selected? Does this violate the majority criterion?

- Conic, yes
- Conic, no
- Integral, yes
- Integral no
- Polynomial, yes
- Summation, no

5. A group of 100 business owners in a city are voting on the president of the chamber of commerce. They are choosing from the following four candidates: Harris, Garcia, Miller, and Young. The voting results are summarized below.

	29	17	23	31
1st	Harris	Garcia	Miller	Young
2nd	Garcia	Miller	Harris	Harris
3rd	Young	Harris	Young	Miller
4th	Miller	Young	Garcia	Garcia

Which candidate is the winner using the majority criterion?

- There is no winner
- Harris
- Garcia
- Miller

6. A small town needed to elect a new constable. The town had 300 eligible voters and asked the voters to rank their favorite candidate from one to four, three times. The candidates are Clark, Thomas, Walker, and Wilson. The results of the rankings were:

	19	24	102	155
1st	Clark	Thomas	Thomas	Walker
2nd	Wilson	Clark	Clark	Clark
3rd	Thomas	Wilson	Wilson	Wilson
4th	Walker	Walker	Walker	Thomas

Which candidate would be the winner using the Borda count method? In this particular instance, does the Borda count method violate the majority criterion?

- Clark, yes
- Clark, no
- Thomas, no
- Walker, yes
- Walker, no
- Wilson, no

Answers

1. The majority criterion states if a choice has a majority of first-place votes, that choice should be the winner. Since there were a total of 200 votes cast, a candidate would need to receive at least 101 first-choice votes to win more than half of the votes. None of the candidates reached that threshold, so the winner cannot be determined according to this method.
2. Gus = 47, more than half of the total votes, this does not violate the majority criterion.
3. Aspen wins with 347. Since Aspen has 68 first-choice votes, and the total votes is 100, of them, since there is no difference in the winner, the majority criterion is not violated by using the Borda count here.
4. Integral wins with 728 out of 1275 votes, which is 57% of the vote
5. in the majority criterion, a winner would need 51 first-place votes, since there are a total of 100 votes. No city received 51 votes, so there is no winner.
6. Clark wins with the Borda method with 919 votes. However, only has 19 first-choice votes, where Walker has 155 votes, which is 51.7% of the vote as there are 300 total votes

17.4.1 Apportionment Methods by Calculating Quotas

Determine the standard quota for a state given its population

1. Given the states and populations below, what is the standard quota for state B if there should be a total of 65 representatives? Round to the nearest tenth.

A : 401,000

B : 250,000

C : 75,000

D : 32,000

E : 19,800

2. Given the counties and populations below, what is the standard divisor if there should be a total of 59 representatives? Round your answer to four decimal places.

Monroe: 125,673

Lackawanna: 198,345

Luzerne: 561,287

Wayne: 63,563

Schuylkill: 51,793

3. Given the population data below for four counties in New Hampshire, how many total representatives should there be if the standard divisor is 7,312.8534? Round your answer to the nearest whole number.

Rockingham: 298,000

Cheshire: 76,700

Grafton: 89,200

Carrol: 47,700

Answers

1. 20.9
2. 16,960.3559
3. 70

Calculate the lower and upper quotas for a state given its population

1. Using the information on and populations below, what is the lower quota for State C if there should be a total of 91 representatives?

State	Population
<i>A</i>	89,002
<i>B</i>	340,100
<i>C</i>	123,789
<i>D</i>	639,900

2. Using the information on states and populations below, what is the correct upper and lower quota for State C if there should be a total of 62 representatives?

State	Population
<i>A</i>	89,002
<i>B</i>	340,100
<i>C</i>	123,789
<i>D</i>	639,900

Answer

Answers

1. 9

2. lower: 6, upper: 7

Use Hamilton's method of determining apportionment

1. The number of cleaning staff assigned to each floor of an office building is apportioned based on the number of employees working on that floor on a typical day. The building has 35 cleaning staff. The median number of employees working on each floor is given below. Use Hamilton's method to apportion the cleaning staff. How many cleaning staff members are sent to Floor 2?

Floor	1	2	3	4	Total
Office Workers	230	105	95	120	550

2. A team of 4 pirate ships collects 100 gold doubloons. Ship sizes are given below. Use Hamilton's method to apportion the doubloons between the ships. How many doubloons does Ship D receive?

Ship	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	Total
Pirates	124	105	95	120	444

3. Eighty law enforcement officers are engaged to cover a county with 5 towns. The populations of the towns are given below. Use Hamilton's method to apportion the law enforcement officers to the towns. How many are assigned to Town B?

Town	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	Total
Population	5400	6800	4700	1200	8900	27,000

4. Suppose 15 volunteer soccer coaches are distributed amongst 4 age groups according to enrollment, given below. Use Hamilton's method to apportion the volunteers. How many coaches will the 8 – 9 age group have?

Age Group	4 – 5	6 – 7	8 – 9	10 – 11	Total
Enrollment	64	130	92	86	372

Answers

1.

Allocation is worked out in the table below.

Floor	Office Workers	Quota	Initial	Final
1	230	14.63636	14	15
2	105	6.68182	6	7
3	95	6.04545	6	6
4	120	7.63636	7	7

Floor 2 gets 7 cleaning staff members

2. 27

3. 20

4. 4

Use Webster's method of determining apportionment

1. The four counties in the twin cities area of Minnesota (Hennepin, Ramsey, Dakota, and Anoka) share 61 representatives. Their populations are shown in the table below. If the representative seats are apportioned using Webster's method, how many seats will Anoka County receive?

County	Population
Anoka	344,151
Dakota	414,686
Hennepin	1,223,149
Ramsey	538,133

2. Members of a think tank are setting up a civics contest to study apportionments. The table below shows the populations of four fictitious states in a country.

State	Population (thousands)
<i>A</i>	37
<i>B</i>	62
<i>C</i>	49
<i>D</i>	94

These four states will share 150 seats in the governmental assembly, apportioned using Webster's method. How many seats are apportioned to State B?

3. A new state has four counties: A, B, C, and D . Their populations are shown in the table below.

County	Population (thousands)
<i>A</i>	236
<i>B</i>	311
<i>C</i>	289
<i>D</i>	194

If Webster's method is used to apportion 113 representatives among these four counties, what standard divisor would be used to find the standard quota for each county? Round your answer to three decimal places.

4. The table shows the populations of the four smallest emirates in the United Arab Emirates. These four emirates share 118 seats in the parliament assembly.

Emirate	Population
Sharjah	793, 573
Ajman	206, 997
Um Al-Awain	49, 159
Fujairah	125, 698

5. A newly formed country has four states: A, B, C, and D. Their populations are shown in the table below.

State	Population (thousands)
<i>A</i>	75
<i>B</i>	109
<i>C</i>	44
<i>D</i>	27

The governors of these states decide that the 63 representatives the states share will be apportioned using Webster's method. What is the sum of the initial rounded quotas for the four states?

6. A country has four states: L, M, N, and P. The state populations are shown in the table below.

State	Population (thousands)
L	172
M	264
N	142
P	228

If Webster's method is used to apportion 90 representatives among these states, how many combined representatives will be assigned to States M and N?

Answers

1.

County	Standard Quota	Rounded Quota
Anoka	8.330	8
Dakota	10.038	10
Hennepin	29.607	30
Ramsey	13.026	13

Anoka County is assigned 8 representatives.

2. 39

3. 9.115

4. 13

5. 64

6. 46

Use Jefferson's method of determining apportionment

1. In the table below, four province populations are shown. Use Jefferson's method to apportion 18 delegates to the provinces. How many delegates come from Province A.

Province	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	Total
Population	76,000	95,000	96,000	89,000	356,000

2. Gift cards are distributed to the 6 top salespeople at the end of the year, according to the number of sales made during the past year. Sales over the last year are given below. Use Jefferson's method to apportion 35 gift cards. How many gift cards are awarded to Salesperson C?

Salesperson	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	Total
Sales (in thousands of \$)	830	640	590	540	490	480	3570

3. A county has 5 towns which are represented at the county seat on a council of 20 delegates. Populations for these towns are given below. Use Jefferson's method to apportion the delegates to the towns. How many delegates does Town C receive?

Town	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	Total
Population	3,764	7,950	5,964	2,898	8,185	28,761

4. Suppose 15 volunteer soccer coaches are distributed amongst 4 age groups according to enrollment, given below. Use Jefferson's method to apportion the volunteers. How many coaches will the 8 – 9 age group have?

Age Group	4 – 5	6 – 7	8 – 9	10 – 11	Total
Enrollment	64	130	92	86	372

Answers

1. 4

2. 6

3. 4

4. 4