Homework 8 – Section 12A/12B/12C

17.2.1 Voting Methods - Winning by Plurality

Identify elements of a preference table

1. Robinson Elementary School students are ranking their favorite school lunches. The choices are pizza (P), chicken fingers (C), hamburgers (H), and spaghetti (S). The ballots have been combined into the following preference table.

Total
512

	112	63	85	103	51	98
1st choice	P	S	S	P	H	C
2nd choice	S	C	H	C	P	H
3rd choice	C	P	P	H	S	P
4th choice	H	H	C	S	C	S

What percentage of students picked spaghetti as their first choice? Round your answer to one decimal place.

How many students picked pizza as their first choice?

$S_{total} = 63+85 = 148$, $\%S = 148/512 \cdot 100 = 28.9\%$

$P_{total} = 112+103 = 215$ votes

2. Given the preference table below, how many voters have candidate B as their second choice? And candidate D as their third choice

	13	64	19	110	33	15
1st choice	A	B	C	D	A	D
2nd choice	D	C	A	C	B	B
3rd choice	B	D	D	A	D	C
4th choice	C	A	B	B	C	A

B_{total} (second choice) $= 33+15 = 48$ votes

D_{total} (third choice) $= 64+19+33 = 116$ votes

Answers

1. 28.89%, 215 students

2. 48 voters ,116 voters

Determine the winner of an election using the plurality method

1. Suppose the board members of a company are deciding where to move their corporate office. They are considering moving to Florida (F), Maine (M), Arkansas (A), or California (C). The board conducted a poll among the employees of the company. The preference table below shows the results.

	50	40	45	20	30	20
1st	A	M	M	M	F	C
2nd	C	C	C	A	C	M
3rd	M	A	F	C	M	F
4th	F	F	A	F	A	A

Assuming that the board will use the plurality method to determine where to move the office, how many first-choice votes dos the winning option receive?

What is the minimum number of additional votes that California needs to win the election?

Forida (f) = 30

Maine (M) = 40 + 45 + 20 = 105 ✓

Arkansas (A) = 50

California (C) = 20

Maine wins w/ 105 votes

Section 12A/12

2. Consider the preference table below.

	23	12	15	10	15	5
1st	A	A	C	B	C	B
2nd	B	C	A	A	B	C
3rd	C	B	B	C	A	A

Using the plurality method, how many first-choice votes would the third-place finisher need to add to win (without a tie)?

$A_{total} = 23 + 12 = 35$

$B_{total} = 10 + 5 = 15$

$C_{total} = 15 + 15 = 30$

B is 3rd choice, needs 36 votes to win

$36 - 15 = 21$ more votes needed

3. Consider the preference table below for an election.

	26	31	x	17	18	24
1st	A	M	C	M	F	C
2nd	C	C	M	A	C	M
3rd	M	A	F	C	M	F
4th	F	F	A	F	A	A

The election officials will use the plurality method to determine the outcome of the election. What is the smallest value for x that will ensure C will win the election?

$A_{total} = 26$

$M_{total} = 31 + 17 = 48$

$C_{total} = x + 24$

$F_{total} = 18$

$x + 24 = 49$

$x = 25$

Answers

1. Maine wins with 105 votes. California needs 86 more votes to win

2. B needs 21 more votes to win.

~~Both Atlanta and Austin have 85 votes, resulting in a run-off.~~

3 4. x = 25

Determine the winner of an election using the Borda count method

1. Suppose CoffeeMan, LLC is looking for a city in which to locate their headquarters. Consider the voting preference table below.

3 pts
2 pts
1 pt

	7	1	4
1st	Seattle	Emporia	Tacoma
2nd	Tacoma	Tacoma	Emporia
3rd	Emporia	Seattle	Seattle

Using the election data and the Borda count method, how many voting points does the second-place finisher receive?

Seattle

$3(7) + 1(1+4) = 21 + 5 = 26$

Emporia

$3(1) + 2(4) + 1(7) = 18$

Tacoma

$3(4) + 2(7+1) = 28$

1st - Tacoma (28 pts)

2nd - Seattle (26 pts)

3rd - Emporia (18 point(s)

2. Based on the preference table below, how many voting points does the winning candidate receive, using the Borda count method?

3 pts
2 pts
1 pts

	32	41	50
1st	*A*	*B*	*C*
2nd	*B*	*C*	*A*
3rd	*C*	*A*	*B*

A

$3(32) + 2(50) + 1(41) = 237$

B

$3(41) + 2(32) + 1(50) = 237$

C

$3(50) + 2(41) + 1(32) = 264$

C wins with 264 points

3. Consider the preference table below. Which list shows the candidates, A, B, C, or D, in the correct order of voting points from highest number of points to lowest number of points, under the Borda count method?

	21	12	6	47	9
1st	A	B	A	D	A
2nd	C	C	C	C	B
3rd	B	D	D	A	D
4th	D	A	B	B	C

4pts
3pts
2pts
1pt

- (C, D, A, B)
- (D, C, A, B)
- (A, C, D, B)
- (D, A, C, B)

C

3(21+12+6+47)+1(9) = 267

D

4(47)+2(12+6+9)+1(21) = 2

A

4(21+6+9)+2(47)+1(12) = 250

B

4(12)+3(9)+2(21)+1(6+47) = 170

1st- C (267 pts)
2nd - D (263 pts
3rd - A (250 pts)
4th - B (170 pts)

4. Consider the preference table below. If the Borda count method is used to determine the winner, how many voting points does the last-place finisher have in this election?

	30	12	6	25
1st	Ronny	Jill	Ronny	Carlos
2nd	Jill	Carlos	Carlos	Ronny
3rd	Bobby	Bobby	Jill	Bobby
4th	Carlos	Ronny	Bobby	Jill

4pts
3pts
2pts
1pts

Ronny

4(30+6)+3(25)+1(12) = 231

Jill

4(12)+3(30)+2(6)+1(25) = 175

Carlos

4(25)+3(12+6)+1(30) = 184

Bobby

2(30+12+25)+1(6) = 140

1st - Ronny (231 pts)
2nd - Carlos (184 pts)
3rd - ~~Jill (175 pts)~~
4th - Bobby (140 pts)

5. Consider the preference table below. Which candidate, A, B, C, or D wins the following election if the Borda count method is used?

	21	12	6	47	9
4pts 1st Choice	A	B	A	D	A
3pts 2nd Choice	C	C	C	C	B
2pts 3rd Choice	B	D	D	A	D
1pt 4th Choice	D	A	B	B	C

A

4(21+6+9)+2(47)+1(12) = 250

B

4(12)+3(9)+2(21)+1(6+47) = 170

C

3(21+12+6+47)+1(9) = 267

D

4(47)+2(12+6+9)+1(21) =

1st_C(267pts)
2nd_D(263pts)
3rd_A(250pts)
4th_B(170pts)

6. Suppose four people are running for positions in the Graduate Student Council. The council will decide the election using the Borda count method. Consider the preference table below.

	21	12	6	47
4pts 1st	Jill	Bobby	Ronny	Carlos
3pts 2nd	Ronny	Carlos	Carlos	Ronny
2pts 3rd	Bobby	Jill	Jill	Bobby
1pts 4th	Carlos	Ronny	Bobby	Jill

The two candidates who receive the most voting points will be on the council. Which two students will be elected?

- ☐ Bobby
- ☑ Carlos
- ☐ Jill
- ☑ Ronny

Jill

4(21)+2(12+6)+1(47) = 167

Bobby

4(12)+2(21+47)+1(6) = 190

Ronny

4(6)+3(21+47)+1(12) = 240

Carlos

4(47)+3(12+6)+1(21) = 263

1st_Carlos(263pts)
2nd_Ronny(240pts)

Answers

1. 26 points

2. 264 points

3. (C, D, A, B)

4. 140 points

5. candidate C

6. Carlos and Ronny

17.2.2 Comparison Voting Methods

Decide the winner of an election using the plurality with elimination method

1. Consider the voting preference table below. use plurality with elimination method

	21	42	36	47	29	51
1st	A	B	A	D	A	C
2nd	D	C	C	C	B	B
3rd	B	D	D	A	D	D
4th	C	A	B	B	C	A

Total
226

majority is
$\frac{226}{2} + 1 = 114$

Step 1
A
21+36+29=86
B
42
C
51
D
47
No majority

Step 2 eliminate B
A
86
C
51+42=93 (No majority)
D
47

Step 3 eliminate D
A
86
C
93+47=140

C wins with 140 votes

2. Consider the voter preference table below.

	23	12	15	10	20	9
1st	Ally	Twyla	Twyla	Jon	Bill	Jon
2nd	Jon	Jon	Ally	Twyla	Twyla	Ally
3rd	Bill	Ally	Bill	Bill	Jon	Twyla
4th	Twyla	Bill	Jon	Ally	Ally	Bill

Total
89

Majority
$\frac{89}{2} + 1 = 45$
$= 45$ vot

Using the plurality with elimination method, which of the following situations would produce a majority winner in the second round?

- Ally receives 20 more first-choice votes and no other additional votes exist
- Bill receives 25 more first-choice votes and no other additional votes exist
- None of these choices
- Twyla receives 16 more first-choice votes and no other additional votes exist

1st Round

Ally
23
Twyla
12+15=27
Jon
10+9=19
Bill
20
No majority
eliminate Jon

2nd Round

Ally
23+9=32
Twyla
27+10=37
Bill
20

1st situation

Adding 20 to ally
32+20=52
Now, majority = $\frac{89+20}{2}$ +
= 55 vote
Not a majority

2nd situation

Add 25 to Bill
20+25=45
Now, majority = $\frac{89+25}{2}$ +1
= 58 votes
Not a majority

3rd situation

Add 16 to Twyla
37+16=53
Now majority = $\frac{89+16}{2}$ +1
= 53 votes

Twyla ← wins w/53 votes

3. Consider the voting preferences in the table below.

	23	12	15	10	20	9
1st	Ally	Twyla	Twyla	Jon	Bill	Jon
2nd	Jon	Jon	Ally	Twyla	Twyla	Ally
3rd	Bill	Ally	Bill	Bill	Jon	Twyla
4th	Twyla	Bill	Jon	Ally	Ally	Bill

Using the plurality with elimination method, which of the choices leads the election at the end of the second round, and with how many first-choice votes?

- Ally leads at the end of the second round with 40 first-choice votes
- Twyla leads at the end of the second round with 37 first-choice votes
- Ally leads at the end of the second round with 32 first-choice votes
- Twyla leads at the end of the second round with 32 first-choice votes

From earlier (2nd Round)

Ally	Twyla	Bill
32	37	20

4. Consider the voting preference table below.

	21	12	26	47	19	11
1st	A	D	C	C	B	D
2nd	B	A	D	D	D	A
3rd	C	C	A	B	A	B
4th	D	B	B	A	C	C

Total

136

Majority = $\frac{136}{2}$

= 69

Using the plurality with elimination method, how many first-choice votes will the winner have at the end of the last round?

1st Round

A = 21, B = 19

C = 26 + 47 = 73

D = 12 + 11 = 23

C has majority w/73 votes

C wins

5. Consider the preference table below.

Total
12
majority
$= \frac{12}{2} + 1 = 7$

	3	4	5
1st	A	B	C
2nd	B	C	A
3rd	C	A	B

1st Round
A = 3, B = 4
C = 5
No majority
eliminate A

2nd round
B = 4 + 3 = 7
C = 5
B has majority
B wins w/ 7 votes

- A wins in the first round
- B wins in the first round
- C wins in the first round
- A wins in the second round
- B wins in the second round
- C wins in the second round

6. Consider the voting preference table below.

	23	12	15	10	20	9
1st	Ally	Twyla	Twyla	Jon	Bill	Jon
2nd	Jon	Jon	Ally	Twyla	Twyla	Ally
3rd	Bill	Ally	Bill	Bill	Jon	Twyla
4th	Twyla	Bill	Jon	Ally	Ally	Bill

Using the plurality with elimination method, how many first-choice votes will the winner have at the end of the last round?

From earlier (2nd Round) — (majority is 45 votes)

Ally 32 | Twyla 37 | Bill 20

No majority, eliminate Bill

Ally 32 | Twyla 37 + 20 = 57

Twyla wins w/ 57 votes

Answers

1. C wins with 140 votes
2. Twyla receives 16 more first-choice votes and no other additional votes exist
3. Twyla leads at the end of the second round with 37 first-choice votes
4. C receives 73 first-choice votes in the first round, which is a majority
5. B wins in the second round
6. Twyla wins the election in the third round with 57 first-choice votes.

Select the winner of an election using the pairwise comparison method

1. Consider the voter preference table below.

	21	12	6	47	9
1st	A	B	A	D	A
2nd	D	C	C	C	B
3rd	B	D	D	A	D
4th	C	A	B	B	C

- Choice A wins the election
- Choice B wins the election
- Choice C wins the election
- Choice D wins the election
- There is a tie between A and D
- There is a tie between B and C
- There is a three-way tie among B, C, and D

Using the pairwise comparison method, how many more points does D receive than B?

A-B
A-C
A-D
B-D
B-C
C-D
6 pairings

D = 1+1+1 = 3 pts
C = 1+1 = 2 pts
B = 0 pts
A = 1 pt

A vs B
A = 21+6+9 = 36 _ 1st
= 47 _ 3rd
36+47 = 83
B
12 (only time B beats A)
A gets 1 pt

A vs C
A = 36
C = 12+47 = 59
C gets 1 pt

A vs D
A = 36
D = 47+12 = 59
D gets 1 pt

B vs D
B = 12+9 = 21
D = 47+21+6 = 74
D gets 1 pt

B vs C
B = 12+9+21
C = 6+47 = 53
C gets 1 pt

C vs D
C = 12+
= 18
D = 47+
+9
= 77
D gets 1 pt

A-B
A-C ✓
A-D
B-C ✓
B-D
C-D ✓

2. Consider the preference table below.

	32	54	41	37	28
1st	A	B	B	B	D
2nd	C	C	C	A	C
3rd	B	A	D	C	B
4th	D	D	A	D	A

If the pairwise comparison method is used to determine the election results, how many points does choice C receive?

A-C

A = 32+37

C = 54+41+28
= 123

C gets 1 pt

B-C

B = 54+41+37
= 132

C = 32+28 = 60

B gets 1 pt

C-D

C = 32+54+41+37 = 164

D = 28

C gets 1 pt

C = 1+1 = 2 pts

3. Consider the preference table below.

	3	4	5
1st	A	B	C
2nd	B	C	A
3rd	C	A	B

Use the pairwise comparison method between B and C. How many votes does the winner of this comparison receive?

B wins w/ 7 votes

B-C

B = 4+3 = 7

C = 5

A-B
A-C
A-D for example

4. If an election with four candidates is decided using the pairwise comparison method, which of the following situations is NOT possible?

- A candidate loses with 1 point
- A candidate loses with 2 points
- A candidate wins with 3 points
- A candidate wins with 4 points

six comparisons, but each candidate is only compared w/ 3 cand so most points a cand can get is 3 point

5. Consider the preference table below from an election.

	30	12	6	25
1st	Ronny	Jill	Ronny	Carlos
2nd	Jill	Carlos	Carlos	Ronny
3rd	Bobby	Bobby	Jill	Bobby
4th	Carlos	Ronny	Bobby	Jill

R-J
R-C
R-B
J-C
J-B
B-C

If the pairwise comparison method is used to determine the results of the election, which candidate would win?

- Bobby wins the election
- Jill wins the election
- Ronny wins the election
- Ronny and Jill tie
- Bobby and Carlos tie
- There is a three-way tie among Carlos, Jill, and Ronny
- There is a three-way tie among Bobby, Jill, and Ronny

R = 1+1 = 2
C = 1+1 = 2
J = 1+1 = 2
B = 0

R-J
R = 30+6+25
= 61
J = 12
1 point for Ronny

R-B
R = 30+6+25
= 61
B = 12
Ronny gets 1 pt

J-B
J = 12+30+6
= 48
B = 25
J gets 1pt

R-C
R = 30+6 = 36
C = 25+12 = 37
Carlos sets 1 pt

J-C
J = 12+30 = 42
C = 25+6 = 31
Jill sets 1 pt

B-C
B = 30
C = 25+12+6
= 43
Carlos gets 1 pt

6. Consider the voter preference table below.

	12	13	17	21	27	11
1st	A	B	E	C	A	C
2nd	D	E	A	D	E	E
3rd	B	C	C	A	B	D
4th	C	D	D	B	D	A
5th	E	A	B	E	C	B

A-B C-D
A-C ✓ C-E
A-D D-E
A-E
B-C ✓
B-D
B-E

Using the pairwise comparison methods, how many points (from winning pairwise comparisons) does C have?

A-C

A=12+27+17.
=56

C=21+11+13
=45

A wins
No pts
for C

B-C

B=13+12+27
=52

C=21+11+17
=49

B wins
no pts
For C

C-D

C=21+11+13
+17=62

D=12+27
=39

C gets
1 pt

C-E

C=21+11+12=44

E=17+13+27=57

E wins, no
pts for C

C gets 1 pt

Answers

1. Choice D wins the election, and has 3 more points then B since B has no votes

2. B wins with 3 points, and C came in second with 2 points

3. 7 votes

4. In a four way race, say between A,B,C, and D, there are six pairwise comparisons, AB, AC, AD, BC, BD, and CD. Each candidate is compared with three other candidates. The maximum number of times a candidate can win in a comparison is 3 times. Therefore, a candidate cannot receive more than 3 points.

5. There is a three-way tie among Carlos, Jill, and Ronny

6. In total, C earned 1 point

17.3.1 Criterion of Voting Methods - Majority, Head to Head count, and Monotonicity

Determine a voting systems fairness using the majority criterion

1. A group of 200 leaders in a city are voting on a temporary replacement for the mayor. Leaders are choosing from the following four candidates: Jones, Brown, Lewis, and Davis. Four separate votes for the temporary mayor were as follows:

Total
200
majority
$\frac{200}{2}+1=101$

	37	43	81	39
1st	Jones	Lewis	Davis	Jones
2nd	Brown	Jones	Jones	Lewis
3rd	Lewis	Brown	Lewis	Brown
4th	Davis	Davis	Brown	Davis

Which candidate is the winner using the majority criterion?

- Lewis
- Davis
- Brown
- Jones
- Cannot be determined

Jones = 37+39 = 76
Lewis = 43
Davis = 81
Brown = 0

NO majority

2. A person gave a dog to an animal rights group. The group is deciding upon a name, choosing from the following four names: Pox, Zac, Gus, and Kid. The results from four rounds of preferences for the name were:

	12	14	38	15	9
1st	Pox	Kid	Gus	Pox	Gus
2nd	Zac	Gus	Kid	Gus	Gus
3rd	Gus	Pox	Zac	Kid	Zac
4th.	Kid	Zac	Pox	Zac	Kid

Total = 88
majority
$\frac{88}{2}+1=45$

Using the plurality method, which name will be selected? Does this violate the majority criterion?

- Gus, no
- Gus, yes
- Pox, no
- Pox, yes
- Kid, yes
- Zac, yes
- Zac, no

Pox = 12+15 = 27
Kid = 14
Gus = 38+9 = 47
Zac = 0
Gus wins with 47 > 45 the majority

3. A group of scientists is getting together for a conference. They are deciding upon a location, choosing from the following four cities: Aspen, Omaha, Butte, and Tulsa. The votes from four rounds of voting for where to hold the conference were as follows:

	53	21	11	15
1st	Aspen	Tulsa	Omaha	Aspen
2nd	Omaha	Omaha	Aspen	Tulsa
3rd	Butte	Aspen	Tulsa	Butte
4th	Tulsa	Butte	Butte	Omaha

Total
100
majority
$\frac{100}{2}+1=51$

4pts
3pts
2pts
1pt

If winner is selected using the Borda method, who wins? Does this violate majority criteria

- Aspen, yes
- Aspen, no
- Butte, yes
- Omaha, yes
- Omaha, no
- Tulsa, no

Tulsa
4(21)+3(15)+2(11)+53
= 204

Omaha
4(11)+3(53+21)+15
= 281

Butte
2(53+15)+21+11 = 168

Aspen wins

1st choice votes
53+15 = 68

which is majority

Aspen
4(53+15)+3(11)+2(21)
= 347

4. A state math society is voting on the main design of a new flag. The flag committee has narrowed the designs to four: Polygon, Summation, Integral, or Conic. A ballot was mailed out to the membership four times trying to determine the design the most members wanted. The results of the rankings are summarized below.

	165	382	431	297
1st	Polynomial	Conic	Integral	Integral
2nd	Summation	Polynomial	Summation	Polynomial
3rd	Conic	Summation	Conic	Summation
4th	Integral	Integral	Polynomial	Conic

Total
1275
majority
$\frac{1275}{2}+1=638$

Using the plurality method, which design will be selected? Does this violate the majority criterion?

- Conic, yes
- Conic, no
- Integral, yes
- Integral no
- Polynomial, yes
- Summation, no

Polynomial 165 | Conic 382 | Integral 431+297=728 | Summation 0

Integral wins

5. A group of 100 business owners in a city are voting on the president of the chamber of commerce. They are choosing from the following four candidates: Harris, Garcia, Miller, and Young. The voting results are summarized below.

	29	17	23	31
1st	Harris	Garcia	Miller	Young
2nd	Garcia	Miller	Harris	Harris
3rd	Young	Harris	Young	Miller
4th	Miller	Young	Garcia	Garcia

Total
100
majority
$\frac{100}{2}+1=51$

Which candidate is the winner using the majority criterion?

- There is no winner
- Harris
- Garcia
- Miller

Harris 29 | Garcia 17 | Miller 23 | Young 31

6. A small town needed to elect a new constable. The town had 300 eligible voters and asked the voters to rank their favorite candidate from one to four, three times. The candidates are Clark, Thomas, Walker, and Wilson. The results of the rankings were:

	19	24	102	155
1st	Clark	Thomas	Thomas	Walker
2nd	Wilson	Clark	Clark	Clark
3rd	Thomas	Wilson	Wilson	Wilson
4th	Walker	Walker	Walker	Thomas

Which candidate would be the winner using the Borda count method? In this particular instance, does the Borda count method violate the majority criterion?

- Clark, yes
- Clark, no
- Thomas, no
- Walker, yes
- Walker, no
- Wilson, no

Answers

1. The majority criterion states if a choice has a majority of first-place votes, that choice should be the winner. Since there were a total of 200 votes cast, a candidate would need to receive at least 101 first-choice votes to win more than half of the votes. None of the candidates reached that threshold, so the winner cannot be determined according to this method.

2. Gus = 47, more than half of the total votes, this does not violate the majority criterion.

3. Aspen wins with 347. Since Aspen has 68 first-choice votes, and the total votes is 100, of them, since there is no difference in the winner, the majority criterion is not violated by using the Borda count here.

4. Integral wins with 728 out of 1275 votes, which is 57% of the vote

5. in the majority criterion, a winner would nee 51 first-place votes, since there are a total of 100 votes. No city received 51 votes, so there is no winner.

6. Clark wins with the Borda method with 919 votes. However, only has 19 first-choice votes, where Walker has 155 votes, which is 51.7% of the vote as there are 300 total votes

17.4.1 Apportionment Methods by Calculating Quotas

Determine the standard quota for a state given its population

1. Given the states and populations below, what is the standard quota for state B if there should be a total of 65 representatives? Round to the nearest tenth.

$A: 401,000$
$B: 250,000$
$C: 75,000$
$D: 32,000$
$E: 19,800$

Total: 777,800

$(250/777.8)(65) \approx 20.8922$

$= 20.9$

2. Given the counties and populations below, what is the standard divisor if there should be a total of 59 representatives? Round your answer to four decimal places.

Monroe: 125,673
Lackawanna: 198,345
Luzerne: 561,287
Wayne: 63,563
Schuylkill: 51,793

Total: 1,000,661

$\text{Std Divisor} = \frac{\text{population}}{\#\text{ reps}}$

$= \frac{1,000,661}{59} = 16,960.3559$

3. Given the population data below for four counties in New Hampshire, how many total representatives should there be if the standard divisor is 7,312.8534? Round your answer to the nearest whole number.

Rockingham: 298,000
Cheshire: 76,700
Grafton: 89,200
Carrol: 47,700

Total = 511,600

$\#\text{ reps} = \frac{\text{pop}}{\text{std divior}}$

$= \frac{511,600}{7,312.8534} \approx 69.959$

$= 70$ reps

Answers

1. 20.9
2. 16, 960.3559
3. 70

Calculate the lower and upper quotas for a state given its population

1. Using the information on and populations below, what is the lower quota for State C if there should be a total of 91 representatives?

State	Population
A	$89,002$
B	$340,100$
C	$123,789$
D	$639,900$

Total
1,192,791

std division = $\frac{\text{pop}}{\text{\# reps}}$ = $\frac{1,192,791}{91}$ = 13,107.5 7341

std quota for C = $\frac{123,789}{13,107.7341} \approx 9.44067$

lower: 9

upper: 10

2. Using the information on states and populations below, what is the correct upper and lower quota for State C if there should be a total of 62 representatives?

State	Population
A	$89,002$
B	$340,100$
C	$123,789$
D	$639,900$

Total
1,192,791

$\left(\frac{123,789}{1,192,791}\right) .62 \approx 6.4344197$

upper: 7

lower: 6

Answer

Answers

1. 9

2. lower: 6, upper: 7

Use Hamilton's method of determining apportionment

1. The number of cleaning staff assigned to each floor of an office building is apportioned based on the number of employees working on that floor on a typical day. The building has 35 cleaning staff. The median number of employees working on each floor is given below. Use Hamilton's method to apportion the cleaning staff. How many cleaning staff members are sent to Floor 2?

Floor	1	2	3	4	Total
Office Workers	230	105	95	120	550

std divisor = $\frac{\text{pop}}{\#\text{rep}} = \frac{550}{35} \approx 15.714285\overline{71}$

		Quota	Initial	Final
1	230	14.6366	14	15 ← Floor 2
2	105	6.6819	6	7
3	95	6.0454	6	6
4	120	7.6364	7	7
			Total 33	Total 35

Need 2 more

2. A team of 4 pirate ships collects 100 gold doubloons. Ship sizes are given below. Use Hamilton's method to apportion the doubloons between the ships. How many doubloons does Ship D receive?

Ship	A	B	C	D	Total
Pirates	124	105	95	120	444

std divisor
$\frac{444}{100} = 4.44$

		Quota	Initial	Final
A	124	27.92793	27	28
B	105	23.648 65	23	24
C	95	21.396396	21	21
D	120	27.027027	27	27 ← ship D

Total 98

Need 2 more

3. Eighty law enforcement officers are engaged to cover a county with 5 towns. The populations of the towns are given below. Use Hamilton's method to apportion the law enforcement officers to the towns. How many are assigned to Town B?

Town	A	B	C	D	E	Total
Population	5400	6800	4700	1200	8900	27,000

std divisor
$\frac{27,000}{80} = 337.5$

		Quota	Initial	Final
A	5400	16	16	16
B	6800	20.1481	20	20 ← Town B
C	4700	13.9259	13	14
D	1200	3.5556	3	4
E	8700	26.37037	26	26

Total 78

Need 2 more

4. Suppose 15 volunteer soccer coaches are distributed amongst 4 age groups according to enrollment, given below. Use Hamilton's method to apportion the volunteers. How many coaches will the 8 – 9 age group have?

Age Group	4 – 5	6 – 7	8 – 9	10 – 11	Total
Enrollment	64	130	92	86	372

std divisor $\frac{372}{15} = 24.8$

		Quota	Initial	Final
4-5	64	2.58064	2	3
6-7	130	5.2419	5	5
8-9	92	3.709677	3	4 ← 8-9 age group
10-11	86	3.4677	3	3
		Total	13	
		Need 2 more		

Answers

1.

Allocation is worked out in the table below.

Floor	Office Workers	Quota	Initial	Final
1	230	14.63636	14	15
2	105	6.68182	6	7
3	95	6.04545	6	6
4	120	7.63636	7	7

Floor 2 gets 7 cleaning staff members

2. 27

3. 20

4. 4

Use Webster's method of determining apportionment

1. The four counties in the twin cities area of Minnesota (Hennepin, Ramsey, Dakota, and Anoka) share 61 representatives. Their populations are shown in the table below. If the representative seats are apportioned using Webster's method, how many seats will Anoka County receive?

County	Population
Anoka	344,151
Dakota	414,686
Hennepin	1,223,149
Ramsey	538,133

std divisor = 2,520,119 / 61 = 41313.426

Total = 2,520,119

	Quota	Rounded
Anoka	8.3302	8 ← Anoka = 8
Dakota	10,0376	10
Hennepin	29.6066	30
Ramsey	13.026	13
	Total	61

2. Members of a think tank are setting up a civics contest to study apportionments. The table below shows the populations of four fictitious states in a country.

std divisor

State	Population (thousands)
A	37
B	62
C	49
D	94

$\frac{242}{150} = 1.613\overline{3}$

Total = 242

These four states will share 150 seats In the governmental assembly, apportioned using Webster's method. How many seats are apportioned to State B?

	Quota	Rounded$_1$	Quota$_2$	Rounded$_2$
A	22,934	23	23.08677	23
B	38.4278	38	38,6859	39
C	30,372	30	30.5744	31
D	58.264	58	58.65	57
		Total 147		Total 152

decrease std divsor

$\frac{242}{151} = 1.6026$

Note: B is rounded

	Quota$_3$	Rounded$_3$
A	22.97758	23
B	38.5	39
C	30.42774	30
D	58.37	58

B = 39

$38.5 = \frac{62}{x}$

std quota

$x = 1.61$

total 150

3. A new state has four counties: A, B, C, and D . Their populations are shown in the table below.

County	Population (thousands)
A	236
B	311
C	289
D	194

std divisor
1030 / 113 = 9,11504

Total 1030

If Webster's method is used to apportion 113 representatives among these four counties, what standard divisor would be used to find the standard quota for each county? Round your answer to three decimal places.

So 9,115

4. The table shows the populations of the four smallest emirates in the United Arab Emirates. These four emirates share 118 seats in the parliament assembly.

Emirate	Population
Sharjah	793,573
Ajman	206,997
Um Al-Awain	49,159
Fujairah	125,698

std divisor
1,175,427 = 9961,245
118

Total 1175,427

	Quota	Rounded$_1$	Quota$_2$	Rounded$_2$
Sharjah	79.666	80	79,49	79
Ajman	20,780	21	20,73	21
Um A-Awain	4,935	5	4,92	5
Fujairah	12.6187	13	12,59	13

Total 119

Total 118

Need smaller divisor
Note: Sharjah is the largest population so can be pulled more than other 5

$79,49 = \frac{793,573}{x}$

$x = 9983,3$

5. A newly formed country has four states: A, B, C, and D. There populations are shown in the table below.

State	Population (thousands)
A	75
B	109
C	44
D	27

std divisor
$\frac{255}{63} = 4.047619$

Total = 255

The governors of these states decide that the 63 representatives the states share will be apportioned using Webster's method. What is the sum of the initial rounded quotas for the four states?

	Quota	Rounded
A	18.529411	19
B	26.929411	27
C	10.870588	11
D	6.670588	7

Total 64 ← sum of initial rounded quotas

6. A country has four states: L, M, N, and P. The state populations are shown in the table below.

State	Population (thousands)
L	172
M	264
N	142
P	228

Total = 806

std divisor $= \frac{806}{90} = 8.9556$

If Webster's method is used to apportion 90 representatives among these states, how many combined representatives will be assigned to States M and N?

	Quota	Rounded	Quota 2	Rounded
L	19.20596	19	19,2197	19
M	29,4789	29	29,5	30 ← 30
N	15,85608	16	15,8674	16 ← +16
				46 total
P	25,45906	25	25,4773	25
Total	89			

M

$29,5 = \frac{264}{x}$

$x = 8,9492$

$\frac{P}{n-0.5} = \frac{806}{89.5} = 9,0056$

L	19,0993	19
M	29,3151	29
N	15,7680	16
P	25,3176	25
	Total	89

Answers

1.

County	Standard Quota	Rounded Quota
Anoka	8.330	8
Dakota	10.038	10
Hennepin	29.607	30
Ramsey	13.026	13

Anoka County is assigned 8 representatives.

2. 39

3. 9.115

4. 13

5. 64

6. 46

Use Jefferson's method of determining apportionment

1. In the table below, four province populations are shown. Use Jefferson's method to apportion 18 delegates to the provinces. How many delegates come from Province A.

Province	A	B	C	D	Total
Population	76,000	95,000	96,000	89,000	356,000

std divisor
356,000 / 18
=19,777,7

	Quota$_1$	Truncated$_1$	Quota$_2$	Truncated$_2$
A	3.8427	3	4	4
B	4.8039	4	5	5
C	4.8539	4	5.0526	5
D	4.5	4	4.6842	4
	Total	15	Total	18

$\frac{95000}{x} = 5$; $x = 19000$.

2. Gift cards are distributed to the 6 top salespeople at the end of the year, according to the number of sales made during the past year. Sales over the last year are given below. Use Jefferson's method to apportion 35 gift cards. How many gift cards are awarded to Salesperson C?

Salesperson	A	B	C	D	E	F	Total
Sales (in thousands of $)	830	640	590	540	490	480	3570

	Quota	Truncated	Quota$_2$	Truncated$_2$
A	8,1373	8	8,6458	8
B	6,2745	6	6,6667	6
C	5,7843	5	6,1458	6 ← C
D	5,2941	5	5,625	5
E	4,8039	4	5,1042	5
F	4,7059	4	5,0	5
	Total	32		Total 35

Need 3 more

Try C

$\frac{590}{x} = 6$

$x = 98,3333$ not enough

For E $(490/98,3333) = 498$

$\frac{490}{x} = 5$ Note E(0.8_), F(0,71)

$x = 98$ C(0.78)

$5 = \frac{490}{x}$ $x = 98$

std

$\frac{357}{35}$

$= 10$

$\frac{3570}{n-0,5} = \frac{3570}{34,5} =$

$= 103,47$

A	8,021	8
B	6,1848	6
C	5,7017	5
D	5,2185	5
E	4,7353	4
F	4,6387	4

Total

3. A county has 5 towns which are represented at the county seat on a council of 20 delegates. Populations for these towns are given below. Use Jefferson's method to apportion the delegates to the towns. How many delegates does Town C receive?

Town	A	B	C	D	E	Total
Population	3,764	7,950	5,964	2,898	8,185	28,761

std divisor = 28761 / 20 = 1438.05

	Quota	Truncated	Quota$_2$	Truncated	Quota$_3$
A	2.6174	2	3	3	2.8887
B	5.5283	5	6.3363	6	6.1012
C	4.1473	4	4.7535	4	4.5771
D	2.01522	2	2.3098	2	2.2241
E	5.6917	5	6.5236	6	6.2818
	Total	18	Total	21	Total

Need 2 more

so A(0.6174) and E(0.6917)

$3 = \frac{3764}{x}$

$x = 1254.6667$

$3 = \frac{3764}{x}$; $3x = 3764$

$5.99 = \frac{7950}{x}$ $5.99x = 7950$;

$8.99x = 11714$

$x = 1303$

so Town C has 4 delegates

4. Suppose 15 volunteer soccer coaches are distributed amongst 4 age groups according to enrollment, given below. Use Jefferson's method to apportion the volunteers. How many coaches will the 8 – 9 age group have?

Age Group	4 – 5	6 – 7	8 – 9	10 – 11	Total
Enrollment	64	130	92	86	372

std divisor
$\frac{372}{15} = 24.8$

	Quota	Truncated	Quota 2	Truncated	Quota 3	Truncated
4-5	2.5806	2	3	3	2,9693	2
6-7	5.2419	5	6.0938	6	6,0314	6
8-9	3.7097	3	4,3125	4	4,2684	4 ← 8-9
10-11	3,4677	3	4.0313	4	3,99	3
	Total	13	Total	17	Total	15

Need 2 more

$3 = \frac{64}{x}, 3x = 64, x = 21.3333$

$3.99 = \frac{86}{x}$

$x = 21.5539$

Answers

1. 4

2. 6

3. 4

4. 4